Chapter 13 - Probability
Sections 1 and 4

Lesson objectives: to learn the different types of probability; when to apply them to solve problems; and how to differentiate between them.

Problem of the day: what are the odds that you will get "tails" when flipping a coin if the previous five tosses have all come up "tails"?

What is probability?

Probability is just a ratio of what happens (an "event") versus the sum total of what could happen.

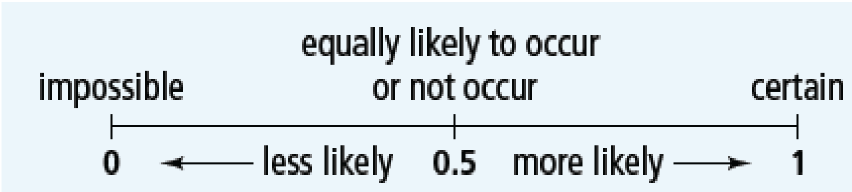
For example, tossing a coin and getting "heads" is 1 out of 2 possible outcomes. So the probability of getting heads is 1:2 or 1/2.

Cutting a deck of cards and getting a 10 has the probability of 4/52. (There are 4 "10" cards out of 52.)

Definition

If the outcomes in a sample space are equally likely to occur, the **probability** of an event P(event) is a numerical value from 0 to 1 that measures the likelihood of an event.

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$$



take note

Key Concept Experimental Probability

Experimental probability of an event measures the likelihood that the event occurs based on the actual results of an experiment.

$$P(\text{event}) = \frac{\text{number of times the event occurs}}{\text{number of times the experiment is done}}$$

Later this week we will conduct experiments. We will calculate the experimental probability from the results of those experiments.

Problem 1 Calculating Experimental Probability

Quality Control A quality control inspector samples 500 LCD monitors and finds defects in three of them.

A What is the experimental probability that a monitor selected at random will have a defect?

$$P(\text{defect}) = \frac{\text{number of monitors with a defect}}{\text{number of monitors inspected}}$$
$$= \frac{3}{500}$$
$$= 0.006 \text{ or } 0.6\%$$

The experimental probability that a monitor selected at random is defective is 0.6%.

B If the company manufactures 15,240 monitors in a month, how many are likely to have a defect based on the quality inspector's results?

$$\text{number of defective monitors} = P(\text{defect}) \cdot \text{total number of monitors}$$
$$= 0.006 \cdot 15{,}240$$
$$= 91.44$$

It is likely that approximately 91 monitors are defective.

Once we know the experimental probability we can use it to estimate future results.

Q: If a baseball team wins 10 out of its first 16 games, what is a possible outcome for 163 games based on the experimental data?

A: you can predict that they will win about 102 games based on that data:

$$\frac{10}{16} = \frac{x}{163}$$

$$101.875 = x$$

Q: based on the previous example, what is the probability that the team will win a randomly selected game?

A: 10/16 or 62.5%

When a sample space consists of real data, you can find the experimental probability. **Theoretical probability** describes the likelihood of an event based on mathematical reasoning.

For example, you can do an experiment by flipping a coin ten times. Theoretically you would predict half heads and half tails or a 50% chance of either one. However, in your experiment, you may get six heads and four tails, making your experimental probability 60% for heads (6/10) and 40% for tails (4/10).

Problem 2 Calculating Theoretical Probability

What is the probability of rolling 7 on a pair of dice?

Step 1 Make a table of the possible results for the rolls of two number cubes. Circle the ones that sum to 7.

Step 2 Find the number of possible outcomes for the event that the sum of two cubes is 7.

Step 3 Find the probability.

$P(\text{rolling a sum of } 7) = \frac{6}{36}$

The probability of rolling numbers that add to 7 is $\frac{6}{36}$, or $\frac{1}{6}$.

	1	2	3	4	5	6
1	1,1	2,1	3,1	4,1	5,1	6,1
2	1,2	2,2	3,2	4,2	5,2	6,2
3	1,3	2,3	3,3	4,3	5,3	6,3
4	1,4	2,4	3,4	4,4	5,4	6,4
5	1,5	2,5	3,5	4,5	5,5	6,5
6	1,6	2,6	3,6	4,6	5,6	6,6

take note

Key Concept Probability of a Complement

The sum of the probability of an event and the probability of its complement is 1.

$P(\text{event}) + P(\text{not event}) = 1 \qquad P(\text{not event}) = 1 - P(\text{event})$

For example, with a single die the probability of rolling 3 or more is 4/6 or 2/3.

Therefore the probability of rolling less than 3 is 1 - 2/3 or 1/3. This is because the probability of the two events must sum to 1! (100%)

A jar contains 10 red marbles, 8 green marbles, 5 blue marbles, and 6 white marbles. What is the probability that a randomly selected marble is not green?

$P(\text{not green}) = 1 - P(\text{green})$	Probability of the complement
$= 1 - \frac{8}{29}$	Find $P(\text{green})$.
$= \frac{21}{29}$	Simplify.

Independent versus Dependent events

A compound event is an event that is made up of 2 or more events.

Essential Understanding You can find the probability of compound events by using the probability of each part of the compound event.

If the occurrence of an event does not affect how another event occurs, the events are called **independent events**. If the occurence of an event does affect how another event occurs, the events are called **dependent events**. To calculate the probability of a compound event, first determine whether the events are independent or dependent.

Problem 1 Identifying Independent and Dependent Events

Are the outcomes of each trial independent or dependent events?

A Choose a number tile from 12 tiles. Then spin a spinner.

The choice of number tile does not affect the spinner result. The events are independent.

B Pick one card from a set of 15 sequentially numbered cards. Then, without replacing the card, pick another card.

The first card chosen affects the possible outcomes of the second pick, so the events are dependent.

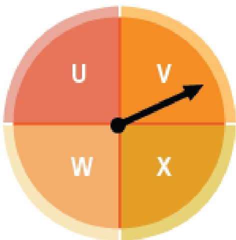
take note

Key Concept Probability of A and B

If A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

The probability that both event A and event B will occur is the probability that A occurs multiplied by the probability that event B occurs.

You roll a standard number cube and spin the spinner at the right. What is the probability that you roll a number less than 3 and the spinner lands on a vowel?



P(A) = 1/3
P(B) = 1/4
P(A and B) = 1/3 * 1/4 = 1/12

take note **Key Concept** **Probability of Mutually Exclusive Events**

If A and B are mutually exclusive events, then $P(A \text{ and } B) = 0$,

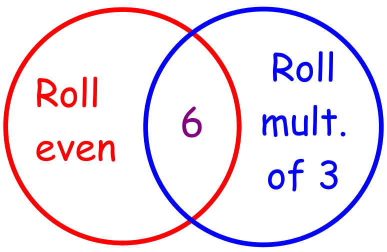
and $P(A \text{ or } B) = P(A) + P(B)$.

For example, the probability that you are in both Math and English is zero. The probability that you are in either Math or English is 1/7 + 1/7 = 2/7

take note **Key Concept** **Probability of Overlapping Events**

If A and B are overlapping events, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

Overlapping events have outcomes in common. For example, for a standard number cube, the event of rolling an even number and the event of rolling a multiple of 3 overlap because a roll of 6 is a favorable outcome for both events.



$$P(\text{even or multiple of 3}) = P(\text{even}) + P(\text{multiple of 3}) - P(\text{even and multiple of 3})$$

$$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6}$$

$$= \frac{4}{6}, \text{ or } \frac{2}{3}$$

Exit Slip

Q1: Is flipping a coin an independent or dependent event?

Q2: For a randomly selected class, what is the probability that you are in a class before fifth period?

Q3: What is the probability that you will roll either a 4 or a multiple of 2 on a die?

Do your math XL (sections 1 and 4)!